

21/1/2015

السبت

د. محمد بن عبد الله

محاضرة [8]

remember:  ${}^nC_r = {}^nC_{n-r}$ 

Example:

If a coin is tossed five times, assuming that the probability of getting ahead in any trial is  $1/3$ , find the probability of getting

i) exactly 3 heads    ii) at least 3 heads

iii) at most 3 heads

$$i) 3h \Rightarrow P(3; 5, \frac{1}{3}) = \binom{5}{3} p^3 q^2$$

$$= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 = 10 \left(\frac{1}{3}\right)^5 \cdot 4$$

$$= \frac{40}{243}$$

$$ii) \text{ at least } 3h \Rightarrow P = P(3h) + P(4h) + P(5h)$$

$$= \binom{5}{3} p^3 q^2 + \binom{5}{4} p^4 q^1 + \binom{5}{5} p^5 q^0$$

$$= \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 + 5 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) + 1 \left(\frac{1}{3}\right)^5$$

$$= \frac{40}{243} + \frac{10}{243} + \frac{1}{243} = \frac{51}{243}$$

$$iii) \text{ at most } 3h \Rightarrow P = P(3h) + P(2h) + P(1h) + P(0)$$

$$= \frac{40}{243} + \binom{5}{2} p^2 q^3 + \binom{5}{1} p^1 q^4 + \binom{5}{0} p^0 q^5$$

$$= \frac{40}{243} + \frac{5 \cdot 4}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3 + 5 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^4$$

$$+ \left(\frac{2}{3}\right)^5 = \frac{1}{243} [40 + 80 + 80 + 32]$$

$$= \frac{232}{243}$$



analogous to the previous example:-

$$\begin{aligned} \text{i) exactly } 2T &= P(2; 5, \frac{2}{3}) \\ &= \binom{2}{5} \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3 = \frac{40}{243} \end{aligned}$$

$$\begin{aligned} \text{ii) almost } 2T &= P(2T) + P(1T) + P(0) \\ &= \dots + \dots + \dots = \frac{51}{243} \end{aligned}$$

$$\begin{aligned} \text{iii) at least } 2T &= P(2T) + P(3T) + P(4T) \\ &\quad + P(5T) = \frac{232}{243} \end{aligned}$$

---

Properties of the binomial distribution:-

Let  $X$  be a random variable with the binomial distribution  $P(K; n, p)$ , then

i - Mean  $= \mu = m = np$

ii - Variance  $= \sigma^2 = npq$

iii - Standard deviation  $= \sigma = \sqrt{npq} = \text{S.D.}$

$$\begin{aligned} \text{mean} = E(x) &= \sum_{x=0}^n x \cdot p(x) \\ &= \sum_{k=0}^n k \cdot P(k; n, p) = 0 \times \frac{n!}{0!(n-0)!} p^0 q^{n-0} + \\ &\quad + \sum_{k=1}^n \frac{n!}{k!(n-k)!} p^k q^{n-k} \\ &= \sum_{k=1}^n \frac{n(n-k)}{k(k-1)!(n-k)!} p \cdot p^{k-1} q^{n-k} \\ &= np \sum_{k=1}^n \frac{(n-1)!}{(k-1)!(n-k)!} p^{k-1} q^{(n-1)-(k+1)} \end{aligned}$$



Let  $S = k-1$ , and  $N = n-1$

as  $k$  runs through the value 1 to  $n$ ,  $S$  runs through the values 0 to  $N$

$$\begin{aligned} S = k-1 & \text{ at } k=1 \quad \therefore S=0 \\ & \text{ at } k=n \quad \therefore S=n-1=N \end{aligned}$$

we denote that we drop the term  $k=0$ , since its value is zero and we factor out  $np$  from each term

$$\begin{aligned} E(x) &= np \sum_{s=0}^N \frac{N!}{s! [(n-1)-(k-1)]!} p^s q^{n-s} \\ &= np \sum_{s=0}^N \frac{N!}{s! [N-s]!} p^s q^{N-s} = np \sum_{s=0}^N \binom{N}{s} p^s q^{N-s} \\ &= np (p+q)^N = np \end{aligned}$$

---

### Poisson Distribution :-

The probability of Poisson Distribution is defined as

$$P(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \dots$$

where  $\lambda$  is some constant.

Poisson distribution can be regarded as the limit of the binomial distribution when the trials  $n$  becomes very large ( $n \rightarrow \infty$ ) and the probability of success become very small. But  $np$  remains a finite constant  $\lambda$ ,  $q = 1 - p$ .



This countably infinite distribution appears in many natural phenomena, such as the number of telephone calls per minute at some switch board, the number of misprints per page in a large text, and the number of  $\alpha$  particles emitted by a radioactive substance.

### Properties of poisson distribution

$$\text{mean} = \mu = \lambda$$

$$\text{Variance} = \sigma^2 = \lambda$$

$$\text{S.D.} = \sigma = \sqrt{\lambda}$$

\* example:

For Poisson distribution, Find:

$$i - P(2; 1) = \frac{1 \cdot e^{-1}}{2!} = 0.184$$

$$ii - P(3, \frac{1}{2}) = \frac{(\frac{1}{2})^3 \cdot e^{-0.5}}{3!} = 0.013$$

$$iii - P(2, 0.7) = \frac{(0.7)^2 \cdot e^{-0.7}}{2!} = 0.12$$